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Isospin Coherence and Final-State Scattering of Disoriented Chiral Condensate *

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Abstract

We examine the validity of the notion of the coherent state for pions and the quantum scattering effect in the final state of pion emission. When the number of particles is large, the effect caused by the small but finite mass difference between the neutral and charged pions can add up substantially in the quantum evolution of an initially coherent state. As a result, the states with quite different numbers of neutral or charged pions are essentially *incoherent*. The importance of the quantum scattering in the final-state isospin charge distribution of a disoriented chiral condensate (DCC) is investigated. We find that the scattering effect significantly reduces the spectacular Centauro and anti-Centauro events. The deformation of a charge distribution dP/df predicted by the classical field theory is significant only for a DCC with the size of 10 fm or more.

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I. INTRODUCTION

Recently there has been a growth of interest in the subject of coherent pion radiation, in conjunction with the conjectured disoriented chiral condensate (DCC) [1–4]. It is suggested that the very high energy hadron-hadron or nucleus-nucleus collisions may create an extended region of space-time containing strong-interaction vacuum with a non-standard chiral orientation. In the language of the linear σ-model the order parameter which usually points in the σ direction is presumed to be in some other direction. In the "Baked Alaska" scenario [2], the disoriented vacuum is found in the expanding shell of hot collision debris. Indeed, the numerical simulations of the classical equation of motion show some evidence of growth of long wavelength modes when the expanding system is out of equilibrium, leading to the formation of large isospin correlated domains [3]. When the hot shell hadronizes and breaks up, the disorientation is radiated away in Goldstone modes (pions) of a common isospin. The most natural way to connect the classical wave with a quantum state is the coherent state formalism in the limit of a large number of quanta. Such coherent pulses of semiclassical pion fields would lead to anomalously large fluctuations event-to-event in the ratio of neutral to charged pions produced. Assuming the all cartesian isospin directions are equally probable, one derives the probability (P) distribution [1,2]

$$\frac{dP}{df} = \frac{1}{2\sqrt{f}} \tag{1}$$

with the definition of neutral fraction $f = N_{\pi^0}/N$.

In this paper, we examine the validity of the notion of the coherent state for pions, in particular, the quantum disturbance to coherence in the final state of pion emission. We find that the effect caused by the small but finite mass difference between the neutral and charged pions adds up substantially in the quantum evolution of an initially coherent state of pions. As a result, the states with quite different numbers of neutral or charged pions are essentially *incoherent* due to the different phase shifts! This phenomenon occurs only in the system where the number of isospin carrying particles (e.g., the pions) are large and the

evolution duration of the system is long compared to $\mathrm{O}(1/m_\pi)$. Although the notion of the isospin coherence for a many-pion system can be entirely inadequate, the prediction of the neutral pion distribution $P(f)$ of DCC in (1) is not altered by the phase shift caused only by the mass terms. We shall compute the modification of (1) due to the quantum rescattering effects when the interactions are included.

II. ISOSPIN COHERENCE OF DCC

The simplest quantum description of a DCC state is given in terms of the coherent state of the classical pion and σ fields in the linear σ model. When the DCC of a large isospin correlated domain is created, one considers an ideal limit where there is no spatial dependence of the DCC chiral orientation. Since we expect that only small isospin states are created, we consider for simplicity the $I=0$ state which is constructed by the superposition of the oriented DCC states denoted by $|\theta,\varphi\rangle$ over the isospin direction (θ,φ) [6]:

$$|I=0\rangle_{\mathrm{DCC}} = \int d\Omega(\theta,\varphi)|\theta,\varphi\rangle. \tag{2}$$

The DCC state relaxes to the true vacuum state by radiating the Goldstone modes. Note that the light particles emitted during this period are not the Goldstone modes associated with the true vacuum, but the disoriented vacua, which are in general linear combinations of π and σ (they are "disoriented" Glodstone bosons). The classical σ field evolves as well as the pion field, eventually the $|I=0\rangle_{\mathrm{DCC}}$ state turns into the true vacuum (a constant σ field) plus many outgoing low-energy pions.

If one assumes that all pions radiated from a DCC during the relaxation end up in the same orbital state in the true vacuum, the state to which $|I=0\rangle_{\mathrm{DCC}}$ relaxes can be written in a unique, simple form [5,6]. The projection of this state onto an eigenstate of the pion number operator $\hat{N}=\hat{N}_+ + \hat{N}_- + \hat{N}_0$ is

$$|I=0;N\rangle = \frac{1}{\sqrt{(N+1)!}}(2a_+^\dagger a_-^\dagger - a_0^\dagger a_0^\dagger)^{N/2}|0\rangle, \tag{3}$$

where $|0\rangle$ is the true vacuum in which $\langle\sigma\rangle = f_\pi$, and $a_\pm^\dagger$ and $a_0^\dagger$ are the pion creation operators for $\pi^\pm$ and π^0. The same neutral pion distribution $P(f)$ as in (1) is obtained by expanding (3) in the basis of definite charge states $|N_0\rangle$'s for $f = N_0/N$ in the large N, N_0 limit. Note that $N_+ = N_- = (N - N_0)/2$ for $|N_0\rangle$. As long as one accepts the assumption of *a single orbital wavefunction* for pions, the state $|I = 0; N\rangle$ is not subject to any final-state interaction corrections (up to an overall phase) since it is the only $I = 0$ state made of N pions.

However, such a picture is oversimplified. It is not clear how all final pions in the true vacuum can retain the shape of the original wavefunction after a continuous emission of the time-varying "disoriented" Goldstone modes. These Goldstone modes emitted in a disoriented vacuum background field are different from those of the true vacuum in that they contain the σ's as well, which must turn into pions when the disoriented background changes into the true vacuum surrounding the DCC domain. As a result, more than one orbital wavefunction must be allowed for pions and many other isosinglet N pion states can be constructed. Furthermore, the quantum scattering causes the transitions among those $I = 0$ states, and the coherence among different $|N_0\rangle$ states may be lost.

Even if all final pions end up in the same orbital configuration, the isospin breaking effects will break up the coherence among $|N_0\rangle$'s. The isospin breaking caused by the electromagnetic interactions and the u- and d- quark mass difference is usually ignored in the DCC state and in its time development, which is a good approximation in most cases. However, it can be a problem when one is concerned with the coherence among states $|N_0\rangle$ with different N_0's in $|I = 0; N\rangle$. The $\pi^\pm$ and π^0 actually oscillate with slightly different frequencies because of their electromagnetic mass difference $\Delta m_\pi = m(\pi^\pm) - m(\pi^0) = 4.6$ MeV. Consider the single momentum mode $\mathbf{k} = 0$. $a_+^\dagger a_-^\dagger|0\rangle$ and the terms $a_0^\dagger a_0^\dagger|0\rangle$ acquire a relative phase $\delta(t) = 2\Delta m_\pi t$ due to Δm_π, so that (up to an overall phase)

$$|t; N\rangle = \frac{1}{\sqrt{(N+1)!}}(2a_+^\dagger a_-^\dagger - e^{i\delta(t)}a_0^\dagger a_0^\dagger)^{N/2}|0\rangle. \tag{4}$$

At $t = 4$ fm/c, $\delta(t) \sim 0.2$. Although the phase for each π^0 pair is small, it accumulates with

N_0. The relative phase between the two charge states $|N_0\rangle$ and $|N_0+\Delta N_0\rangle$, differing in the number of neutral pions by ΔN_0, gets amplified by ΔN_0 times and becomes $\delta(t)\Delta N_0$. When N is let to infinity with a fixed $\Delta f = \Delta N_0/N$, this phase goes to infinity and wipes out any coherence between the two states. For finite N and ΔN_0, the initial coherence between $|N_0\rangle$ and $|N_0+\Delta N_0\rangle$ is lost in a characteristic time scale $\tau \sim 1/(\Delta N_0 \Delta m_\pi)$. For $\Delta f = 0.2$ and $N = 50$, for instance, the isospin breaking effect rapidly accumulates in $\tau \simeq 4$ fm/c, leaving only the I_3 as a good quantum number. When the mutual coherence among different $|N_0\rangle$ states in (4) is lost, $\pi^\pm$ and π^0 thereafter behave without the overall isosinglet constraint. However, it is interesting to note that the phase $\delta(t)$ does not affect the classical prediction (1) on the distribution $P(f)$ [7]. Our picture is very close to a purely classical description where the states with definite (classical) isospin orientation are produced incoherently, and the charge ratio N_+/N, N_-/N and N_0/N are identified with the isospin direction of the classical pion field.

III. QUANTUM RESCATTERING EFFECTS

We have argued that after the relaxation of the DCC into the true vacuum, the different charge states $|N_0\rangle$'s should be treated as mutually incoherent. As we shall study how the final state quantum scattering changes the prediction in (1), we must start with a pion-number eigenstate $|N_0\rangle$ instead of the isosinglet state in (3). The quantum scattering processes $\pi^0\pi^0 \leftrightarrow \pi^+\pi^-$ cause the transition between $|N_0\rangle$ and $|N_0 \pm 2\rangle$, which can distort the distribution $P(f)$. One needs to worry if there will be enough quantum scattering of pions to dilute the effect of interest in (1). Put more explicitly, suppose a DCC charge state initially consists of only N π^0's, we would like to calculate how many π^+'s or π^-'s end up in the detector after the quantum scatterings are included.

The relevant quantity for determining the importance of the quantum scattering is the mean free path (l_0) of a physical pion in a cloud of emitted pions at the time (say, $t = 0$) when the DCC decays. If the size of the system (R_D) at the time is much smaller than l_0,

the pions essentially decouple from the system and propagate freely with little interactions. The opposite limit is $R_D \gg l_0$, in which case a pion will encounter many rescatterings before leaving the system and the measured value f ($\simeq 1/3$) has little trace of the original charge fraction f_0. In the general case, we have to conduct a quantitative computation. Since the pions from a DCC are soft in general, the mean free path of collision is much longer than a typical size of DCC. We shall make the approximation that the emitted pions stream out almost freely, i.e. that the collision does not slow down the diffusion of the pion cloud. In this approximation, there is a simple relation between the speed of diffusion ($v_{\rm d}$) and the average relative velocity ($v_{\rm rel}$) for pion collisions. With this simplification (an underestimate), we shall show that the quantum corrections are characterized by a rather simple and intuitive factor $\exp[-4R_D/(3l_0)]$. If the size of the diffusing cloud is a significant fraction of the mean free path, the quantum rescattering is non-negligible and will dilute the Centauro and anti-Centauro events [8].

We are concerned with the evolution of the charge composition of the pion cloud with three charge states denoted by $0, +, -$. For simplicity we assume that the pion density is spatially uniform inside the DCC and the typical momentum is determined by the uncertainty principle $\langle p\rangle \approx 1/R_D$. For the DCC of an uniformly aligned orientation, this is a good estimate, but it is an underestimate for the DCC having domain structure. When the typical energy of pions is low, the relevant reaction is the two-body $\pi\pi$ scattering. The neutral pion state, for instance, is depleted by the annihilation process $\pi^0\pi^0 \to \pi^+\pi^-$ and replenished by the reverse process $\pi^+\pi^- \to \pi^0\pi^0$. Under these assumptions, the time evolution of the numbers of three charge particles can be readily derived from the Boltzmann transport equation:

$$\frac{dN_0}{dt} = -\frac{2\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}N_0^2 + \frac{2\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}N_+N_-, \tag{5}$$

$$\frac{dN_+}{dt} = \frac{\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}N_0^2 - \frac{\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}N_+N_-, \tag{6}$$

$$\frac{dN_-}{dt} = \frac{\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}N_0^2 - \frac{\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}N_+N_-, \tag{7}$$

where the cross section $\sigma_{\pi\pi}$ is defined to be that of $\pi^+\pi^- \to \pi^0\pi^0$ (note that $\sigma(\pi^+\pi^- \to \pi^0\pi^0) = \frac{1}{2}\sigma(\pi^0\pi^0 \to \pi^+\pi^-)$). $V(t)$ is the volume of pion cloud enclosed by the outgoing pion front. $\langle v_{\rm rel}\sigma_{\pi\pi}\rangle$ is the average value of the relative velocity times the cross section. Adding the above three equations gives the total number conservation $dN/dt = 0$ where $N = N_0(t)+N_+(t)+N_-(t)$, which is the consequence of the fact that only two-body reactions are considered in collision terms in the Boltzmann equation. Since initially $N_+(0) = N_-(0)$, the evolution equations guarantee that this remains true at later times. In this case, we obtain a simple analytic solution. Define $f_0 \equiv N_0(0)/N$ and $f(t) \equiv N_0(t)/N$. The solution is

$$f(t) = \frac{(f_0+1) + (3f_0-1)e^{-\mathcal{F}(t)}}{3(f_0+1) - (3f_0-1)e^{-\mathcal{F}(t)}}, \tag{8}$$

where $\mathcal{F}(t)$ determines the time dependence of charge composition due to the final-state scatterings,

$$\mathcal{F}(t) = 2N\int_0^t \frac{\langle v_{\rm rel}\sigma_{\pi\pi}\rangle}{V(t)}dt. \tag{9}$$

Before we estimate the $\mathcal{F}(t)$, some remarks on the general features of (8) are in order. $\mathcal{F}(t)$ is a positive number and $e^{-\mathcal{F}(t)} < 1$. Generally f depends on $\mathcal{F}(t)$ *and* the initial value f_0. It is easy to verify that when $f_0 < 1/3$, $f > f_0$ and when $f_0 > 1/3$, $f < f_0$ at later times ($t > 0$). In other words, if the system initially has fewer π^0's compared to π^+'s or π^-'s, the quantum scattering tends to increase the number of π^0's; if more π^0's, the quantum scattering tends to decrease it. $f_0 = 1/3$ is a special value, in which case the time dependence of f completely disappears and f keeps a constant $f = f_0 = 1/3$. In the classical theory, f_0 ranges from 0 to 1 and the end points correspond to the Centauro and anti-Centauro events. These extreme situations do not occur in the quantum theory. In fact, f can never be equal to 0 or 1 as long as $\mathcal{F}(t) \neq 0$. Since the state of all π^0 or all $\pi^\pm$ is degraded by the charge conversion processes, the quantum rescattering squeezes the allowed range for f from the both ends. The upper and lower limits on f are:

$$f_{\max} = \frac{1+e^{-\mathcal{F}}}{3-e^{-\mathcal{F}}} \quad , \quad f_{\min} = \frac{1-e^{-\mathcal{F}}}{3+e^{-\mathcal{F}}}. \tag{10}$$

The ranges for $f < f_{\rm min}$ and $f > f_{\rm max}$ are excluded by the quantum rescattering. If the rescattering is very strong, the factor $e^{-\mathcal{F}}$ would vanish, and $f_{\rm max}$ and $f_{\rm min}$ converge to a single point $f = 1/3$. Even for relatively weak scatterings of interest, the most spectacular Centauro and anti-Centauro events are diluted to some extent.

We would like to estimate the integral in (9) for a realistic DCC. We assume that the volume $V(t)$ of the pion cloud is spherical and diffuses with a constant velocity $v_{\rm d}$ as

$$V(t) = \frac{4\pi}{3}(R_D + v_{\rm d}t)^3. \qquad (0 < t < \infty) \tag{11}$$

The time dependence of the $\pi\pi$ cross section is negligibly weak as long as the typical momentum is in range of validity of the low energy theorem [9]. We approximate it by the threshold value (by doing so we underestimate the effect). According to the soft-pion theorem, the model-independent low energy cross section is given by

$$\sigma(\pi^+\pi^- \to \pi^0\pi^0) = \frac{s}{32\pi f_\pi^4}\left(1 - \frac{m_\pi^2}{s}\right)^2, \tag{12}$$

whose threshold value is $\sigma_{\pi\pi}|_{\rm threshold} = 9m_\pi^2/128\pi f_\pi^4$. The integration over t in (9) can be done trivially,

$$\mathcal{F}(\infty) = \frac{n(0)\langle v_{\rm rel}\sigma_{\pi\pi}\rangle R_D}{v_{\rm d}}, \tag{13}$$

where $n(0) = N/V(0)$ is the initial *total* pion density at the time when the DCC decays. Clearly, $\mathcal{F}(\infty)$ sensitively depends on the average relative velocity defined in a frame-independent way $v_{\rm rel} = \sqrt{(p_1 \cdot p_2)^2 - m_\pi^4}/E_1E_2$. If the collision occurs frequently during diffusion, the diffusion velocity would be much smaller than the average pion velocity. In the other limit, if the collision rarely occurs, the diffusion takes place as pions stream freely. In the latter case $v_{\rm d}$ is equal to $\langle|\vec{p}|\rangle/E$, and the average relative velocity is calculated by integrating $v_{\rm rel}$ over all relative directions: $\langle v_{\rm rel}\rangle \simeq \frac{4}{3}\langle|\vec{p}|\rangle/E$ in the non-relativistic limit if all pions have approximately common $\langle|\vec{p}|\rangle$. Thus, we obtain

$$\mathcal{F}(\infty) \simeq \frac{4}{3}\frac{R_D}{l_0}, \tag{14}$$

where the mean free path at $t = 0$ is $l_0 = 1/(n(0)\sigma_{\pi\pi})$. The initial pion density is determined by the energy density of the DCC divided by the average pion energy, $n(0) = m_\pi^2 f_\pi^2/m_\pi$ in the soft pion limit. Therefore the mean free path l_0 for the conversion $\pi^+\pi^- \to \pi^0\pi^0$ at the threshold energy is given by $l_0 \simeq 128\pi f_\pi^2/9m_\pi^3 \simeq 27.7$ fm, where $f_\pi = 93$ MeV. It is longer than the typical size of the DCC. As the cloud of quanta diffuses, the ratio of the mean free path to the cloud size increases even further. Therefore, for most DCC's, the diffusion of the pion cloud is closer to a free streaming than to a strongly interacting case. We thus justify the approximation in calculating the diffusion velocity v_d and the average pion velocity. For a DCC with a radius $R_D = 5$fm, $\mathcal{F}(\infty)$ is about 0.24. If the DCC consists of only π^0's, i.e. $f_0 = 1$, then $f(\infty) \simeq 80\%$, i.e. 20% of original π^0's are converted into $\pi^\pm$'s!

We can also calculate the distribution dP/df of the final neutral charge fraction $f = N_0(\infty)/N$. The classical field theory predicts $dP/df_0 = 1/2\sqrt{f_0}$ where $f_0 = N_0(0)/N$ is the initial value at the time when the DCC decays. The distribution in terms of f is given by

$$\frac{dP}{df} = \frac{dP}{df_0} \cdot \frac{df_0}{df} = \frac{1}{2\sqrt{f_0(f)}} \cdot \frac{16e^{-\mathcal{F}}}{[(1+3e^{-\mathcal{F}}) - 3f(1-e^{-\mathcal{F}})]^2}, \tag{15}$$

where df_0/df is computed from (8). The distribution dP/df tends to concentrate more in the range around $f = 1/3$ and fall off faster from one end to the other as f increases, making the distinction from non-DCC events a little less conspicuous. In Fig. 1 we have plotted dP/df for two values: $R_D = 4$ fm and 10 fm. The deformation is not too severe to significantly affect the experimental test of (1) for $R_D = 4$ fm, but severe enough to make us worry for $R_D = 10$ fm. For values larger than $R_D = 10$ fm, the mean free path of collision becomes comparable or even shorter than the DCC size. Our assumption that $\langle v_\mathrm{rel}\rangle \simeq 4v_\mathrm{d}/3$ breaks down and v_d becomes slower than the average pion velocity. In other words, pions emitted stick together through final-state interactions. When this happens, the dilution factor $\mathcal{F}(\infty)$ rises quickly with R_D, making the experimental test of the classical theory prediction difficult.

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Figure Captions

Fig. 1 Comparison of the probability distributions of the neutral pion fraction $f = N_{\pi^0}/N_{\mathrm{total}}$ for different sizes of DCC.